Images as Arrays of Numbers

When a digital image is generated by a CCD chip, it consists of an array of numbers. Each number gives the number of electrons that were counted in each specific pixel.

The information to the left gives the number of electrons counted in a CCD image that consists of 3 rows and 3 columns of pixels. Each pixel is labeled by its row and column number starting in the upper left cell element. For example 'P23' is name of the pixel located in row 2, column 3 of the CCD array.

\[
I = \begin{pmatrix}
10387 & 29876 & 40987 \\
20345 & 30987 & 50328 \\
40923 & 50876 & 70987
\end{pmatrix}
\]

\[
I = \begin{pmatrix}
p_{11} & p_{12} & p_{13} \\
p_{21} & p_{22} & p_{23} \\
p_{31} & p_{32} & p_{33}
\end{pmatrix}
\]

**Problem 1** - How many electrons were counted in pixel P32?

**Problem 2** - Which pixel counted the smallest number of electrons?

**Problem 3** - In which direction is the brightness of the image increasing?

**Problem 4** - What is the average brightness of the pixels in this image?

**Problem 5** - The image was exposed for 0.001 seconds. To two significant figures, what was the brightness of the brightest portion of scene being photographed in electrons/sec?

Space Math

http://spacemath.gsfc.nasa.gov
Problem 1 - How many electrons were counted in pixel $P_{32}$?

Answer: 50,876

Problem 2 - Which pixel counted the smallest number of electrons?

Answer: $P_{11}$

Problem 3 - In which direction is the brightness of the image increasing?

Answer: From the top of the image down towards pixel $P_{33}$ in the lower right corner

Problem 4 - What is the average brightness of the pixels in this image?

Answer:

$$I = \begin{pmatrix} 10387 & 29876 & 40987 \\ 20345 & 30987 & 50328 \\ 40923 & 50876 & 70987 \end{pmatrix}$$

The sum of all 9 pixels is 345,696 so the average is $A = \frac{345,696}{9} = 38,411$

Problem 5 - The image was exposed for 0.001 seconds. To two significant figures, what was the brightness of the brightest portion of scene being photographed in electrons/sec?

Answer: Brightness $= \frac{70,987 \text{ electrons}}{0.001 \text{ seconds}}$

$= 70,987,000 \text{ million electrons/sec.}$

$= 71 \text{ million electrons/sec}$

Note: It is very important to note that image arrays are simply tables of numbers. Although the values of specific pixels can be represented by a notation such as $a_{ij}$, they do not represent mathematical objects called matrices such as having an inverse, or obeying the matrix rules of multiplication. Arrays can only be added and subtracted on a pixel by pixel basis such as $c_{ij} = a_{ij} + b_{ij}$